HYPER VERSION

Received: *February 15, 1999*, Accepted: *February 27, 1999*

AdS$_3$/CFT$_2$ correspondence and space-time $\mathcal{N}$=3 superconformal algebra

Satoshi Yamaguchi and Yukitaka Ishimoto

Graduate School of Human and Environmental Studies, Kyoto University
Yoshida-Nihon-Matsu cho, Sakyo-ku, Kyoto 606-8501, Japan
E-mail: yamaguch@phys.h.kyoto-u.ac.jp, ishimoto@phys.h.kyoto-u.ac.jp

Katsuyuki Sugiyama

Department of Fundamental Sciences Faculty of Integrated Human Studies
Kyoto University, Yoshida-Nihon-Matsu cho, Sakyo-ku, Kyoto 606-8501, Japan
E-mail: sugiyama@phys.h.kyoto-u.ac.jp

Abstract: We study a Wess-Zumino-Witten model with target space $AdS_3 \times (S^3 \times S^3 \times S^1)/\mathbb{Z}_2$. This allows us to construct space-time $\mathcal{N} = 3$ superconformal theories. By combining left-, and right-moving parts through a GSO and a $\mathbb{Z}_2$ projections, a new asymmetric $(\mathcal{N}, \overline{\mathcal{N}}) = (3, 1)$ model is obtained. It has an extra gauge (affine) SU(2) symmetry in the target space of the type IIA string. An associated configuration is realized as slantwise intersecting M5-M2 branes with a $\mathbb{Z}_2$-fixed plane in the M-theory viewpoint.

Keywords: Conformal Field Models in String Theory, Brane Dynamics in Gauge Theories, Conformal and W Symmetry.

JHEP02(1999)026

Contents

1. Introduction

One of the most interesting dualities is the correspondence between supergravity theories (SUGRA) on $AdS_{d+1} \times \mathcal{M}$ and the d-dimensional conformal field theories (CFT). Many intensive researches are in progress in various dimensional cases. In an excellent paper by J. Maldacena [1], enhancements of the rigid supersymmetries are proposed in the near-horizon geometries on anti-de Sitter (AdS) spaces of the SUGRA. These theories in the limits are conjectured to be identified with boundary conformal field theories, for an instance, the 4-dimensional $\mathcal{N} = 4$ susy Yang-Mills theory realized on a boundary of AdS_5 in type IIB SUGRA.

Boundary geometries in other dimensions are also believed to have these enlarged symmetries and a better understanding of these equivalences is expected to have numerous applications to strongly coupled gauge theories and could make clear dynamics in the regions. It might be a realization of the profound problem that the dynamics of the supergravity theories are effectively controlled by large N susy Yang-Mills theory in some limit of large solitonic charges. In order to unify various string dualities, it could give us a clue to formulate some fundamental theory by taking D-branes as fundamental objects. For instances, classical solutions in AdS_4-AdS_7-backgrounds are currently investigated in the context of the M-theory.

But most of the works in this subject are restricted to situations where the classical low energy approximation of the SUGRA is reliable. Also turning on Ramond-Ramond (RR) backgrounds make it difficult to study these AdS/CFT correspondences.

In contrast, the three-dimensional case is special in many respects and the analyses of AdS_3 gravity are lifted to those in a stringy level on this background. It supplies a chance for us to establish this correspondence in quantum levels. First, in this case the associated CFT is two-dimensional and has infinite dimensional local symmetries. That makes it possible for us to obtain several fruitful results through concrete calculations.

Second, string theory on AdS_3 can be defined without turning on RR-fields and should be more amenable to traditional worldsheet methods. In fact by performing an S-duality on the type IIB background with RR-fields, we obtain a perturbative string theory with only NS-NS fields turned on. The conformal invariance on the worldsheet of first quantized superstring leads to a classical solution of SUGRA. That is to say, the AdS_3/CFT_2 correspondence in stringy level is reduced to an equivalence between the worldsheet CFT and the space-time (boundary) CFT. From the point of view of perturbative string theory on AdS_3, Giveon et al. [2] have constructed directly superconformal generators in the space-time CFT in terms of physical vertex operators of the worldsheet CFT. Recently there are some further developments along this line [3, 4, 5, 6, 7, 8, 9, 10, 11].

R-symmetries of space-time susy theories are reflected in isometries of subspaces of the $\mathcal{M}$. Most typical examples are illustrated in "S^3", "$S^3 \times S^3$", "S^5" cases whose isometry groups are SU(2), SU(2) $\times$ SU(2), SO(6) $\cong$ SU(4) respectively. There are space-time "small" $\mathcal{N} = 4$ (2 dim), "large" $\mathcal{N} = 4$ (2 dim), and $\mathcal{N} = 4$ (4dim) superconformal symmetries associated to them. When one reduces isometries of these spaces, less supersymmetric theories can be obtained in space-time. In order to carry out this program, a simple useful method is proposed to divide the subspace in terms of some discrete groups Γ [12]. By applying this method to the 5-dimensional case, that is, $AdS_5 \times S^5/\Gamma$ SUGRA, space-time $\mathcal{N}$ =$0, 1, 2, 4$ theories are obtained approximately in the leading order of large N limits.

In general, two-dimensional case is better understood than other higher dimensional analogues. We hope that the division method will be available in order to construct less ($\mathcal{N} < 4$) superconformal theories for 2-dimensional cases without approximations, namely, in all orders of the large N expansions.

Motivated with this consideration, we focus on the superstring theory on AdS_3 background and intend to examine space-time $\mathcal{N} = 3$ superconformal generators in terms of a Wess-Zumino-Witten (WZW) model. Many considerations have been given for (small/large) $\mathcal{N} = 4$ space-time SCFT in AdS_3 models [2, 9, 8, 11]. But there still remain several uncertain points for space-time less susy models. Our aim is to develop a concrete applicable method to construct less susy models in AdS_3 string. We present $\mathcal{N} = 3$ space-time generators explicitly and investigate associated brane configurations.

The purpose of this paper is to continue this study to establish AdS_3/CFT_2 correspondence in less susy models and to apply it to some additional examples that are of interest in the different contexts.

The paper is organized as follows. In section 2, we shortly review the formulation in GKS's paper and its possible generalizations for direct product target spaces. We also explain the Wakimoto's representations [13] of affine Lie algebra currents and associated WZW models in order to fix our conventions in the paper. We propose a orbifold-type model in the superstring as a new class with an AdS_3 background. Taking a $\mathbb{Z}_2$-division on a product space "$S^3 \times S^3 \times S^1$", we construct space-time $\mathcal{N} = 3$ superconformal theories on its fixed locus, that is, a diagonal "S^3_D" space. It is an extension of the work [2] to less space-time susy theories. By imposing several physical conditions, we can obtain space-time generators in terms of world-sheet operators in the first quantized superstring. The algebra is represented linearly and its global part turns out to be a super Lie algebra OSp(3|2). A bosonic part $\mathrm{SL}(2;\mathbb{R}) \times \mathrm{SU}_D(2)$ is associated with an isometry of the sigma model target space $AdS_3 \times S^3_D$. We elaborate some aspects of this model and comment on a proposed identification. We also present modes of space-time $\mathcal{N} = 3$ SCFT generators towards understandings of the spectra of space-time Fock spaces. As concrete examples, we analyse chiral primary fields in this algebra and construct them explicitly.

Next we combine these left-moving $\mathcal{N} = 3$ SCA and right-moving correspondings in the type IIA superstring. Chiralities of two-sides are opposite because of a GSO projection and the resulting theory turns out to have an exotic $(\mathcal{N}, \overline{\mathcal{N}}) = (3, 1)$ supersymmetry in space-time. In addition to the $\overline{\mathcal{N}} = 1$ Virasoro symmetry, we observe that the right-side part has an extra $\overline{\mathcal{N}} = 1$ affine SU(2) super Lie algebra associated with the isometry of the remaining diagonal S^3_D.

In section 3, we introduce a brane configuration whose near horizon geometry will serve as AdS_3 background for string propagation. The configuration includes $M5$, $M2$ branes and a $\mathbb{Z}_2$ fixed plane. We describe the supergravity solution and its near horizon limit. In fact by shrinking the radius of a longitudinal circle S^1, the geometry in the M-theory becomes the orbifold-type space with a $\mathbb{Z}_2$-action in the type IIA theory explored in section 2. The conformal symmetries are realized on a $\mathbb{Z}_2$-fixed plane in this classical solution. We will also explain relations between radii of the spaces and the levels of algebras proposed in [2].

Section 4 is devoted to conclusions and comments. In appendix A, we collect several conventions for spin operators and cocycle factors.

2. Superstring on $AdS_3 \times (S^3 \times S^3 \times S^1)/\mathbb{Z}_2$

The two-dimensional CFT has infinite dimensional symmetries and is investigated in detail. The relations between AdS_3 gravity and the 2dim CFT are expected to be understood much deeper than those in other dimensions. The isometry group of the AdS_3 space is SO(2, 2), but the asymptotic symmetry of them is enhanced to 2dim boundary conformal algebra. This asymptotic symmetry acts on the background fields evaluated at spatial infinity and leaves them invariant. The well-known example

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is the BTZ black hole [14] in AdS_3 and an associated boundary isometry is generated by two commuting Virasoro algebras [15, 14]. Its central charge is characterized by 3dim Newton constant G_3 and a radius ℓ of AdS_3 as $\frac{3\ell}{2G_3}$ [15, 16]. Also these relations with CFT are not only established in the AdS_3 SUGRA, but also are lifted to those at the level in string theory propagating on AdS_3. In the context of the latter, asymptotic Virasoro generators are constructed by worldsheet operators in the RNS formalisms in [2]. Let us review these shortly and consider possible target spaces allowed by consistency conditions.

2.1 General case

First we restrict ourselves to $\mathcal{N} = 1$ WZW models with target spaces of the direct product type

$$AdS_3 \times \prod_{I=1} G_I \,, \qquad AdS_3 \cong \mathrm{SL}(2;\mathbb{R}) \,.$$

Here G_I's are either compact simple groups or abelian U(1) groups. The dimension of each G_I is d_I and its dual Coxeter number $h_I^\vee$. At the quantum level, $\mathcal{N} = 1$ susy affine Lie algebra is realized and associated level is k_I for G_I. Then central charge c_I is calculated as

$$c_I = \left(\frac{3}{2} - \frac{h_I^\vee}{k_I}\right) d_I \,.$$

(We refer $I = 0$ to an index associated with $G_0 = \mathrm{SL}(2;\mathbb{R})$ group.) A constraint is given as a conformal anomaly free condition that expresses a balance of the central charges amongst ghost parts and matter parts

$$c = \sum_{I=0} \left(\frac{3}{2} - \frac{h_I^\vee}{k_I}\right) d_I = 15 \,. \tag{2.1}$$

There is also a relation about a total dimension

$$10 = \sum_{I=0} d_I \,. \tag{2.2}$$

Similarly the space-time boundary affine algebra is expected to have a central charge $\tilde{c} = 6kp$ with $p = \oint \frac{dz}{2\pi i} \gamma^{-1} \partial\gamma$. The γ is a scalar appearing in the Wakimoto representation [13] of the $\mathrm{SL}(2;\mathbb{R})$ currents. A level of boundary affine Lie algebra $\tilde{k}_I$ is proportional to the level k_I of the associated algebra, $\tilde{k}_I = k_I p$. Thus eqs. (2.1) and (2.2) lead us to relations between two kinds of central charges

$$\frac{6^2}{\tilde{c}} = \sum_{I=1} \frac{h_I^\vee d_I}{\tilde{k}_I} \,, \qquad \sum_{I=1} d_I = 7 \,.$$

We can write down all possible cases satisfying these conditions:

1. $\mathrm{SL}(2;\mathbb{R}) \times U(1)^7 \leftrightarrow AdS_3 \times T^7$
2. $\mathrm{SL}(2;\mathbb{R}) \times \mathrm{SU}(2) \times U(1)^4 \leftrightarrow AdS_3 \times S^3 \times T^4$
3. $\mathrm{SL}(2;\mathbb{R}) \times \mathrm{SU}(2) \times \mathrm{SU}(2) \times U(1) \leftrightarrow AdS_3 \times S^3 \times S^3 \times S^1$.

In case 1, the level of $\mathrm{SL}(2;\mathbb{R})$ algebra is infinite ($k = 0$) and the radius of AdS_3 blows up. It is a decompactifying limit of AdS_3 and corresponds to a flat space-time.

The model in case 2 corresponds to an elaborated near horizon limit of the D1-D5 system. The "small" $\mathcal{N} = 4$ CFT is realized as space-time asymptotic symmetry. The AdS_3/CFT_2 correspondence is investigated in the framework of type IIB superstring in [2]. Here (small) $\mathcal{N} = 4$ superconformal generators are constructed from the worldsheet vertex operators.

Similarly a "large" $\mathcal{N} = 4$ SCFT is constructed in the model in case 3 [9]. The associated space-time central charge $\tilde{c}$ is expressed by levels $\tilde{k}_1$, $\tilde{k}_2$ of two SU(2)s

$$\tilde{c} = \frac{6\tilde{k}_1\tilde{k}_2}{\tilde{k}_1 + \tilde{k}_2},$$

as required in a unitary $\mathcal{N} = 4$ theory.

In the context of worldsheet theory, we have other types of possible models. In the following, we concentrate on a orbifold-type case and analyse its associated space-time superconformal theory.

2.2 $\mathcal{N} = 3$ superconformal algebra

In this paper we propose a orbifold type model, that is, a superstring theory on the $AdS_3 \times (S^3 \times S^3 \times S^1)/\mathbb{Z}_2$ as a new class with an AdS_3 background. We explore it and construct space-time $\mathcal{N} = 3$ superconformal algebra.

The string metric of the AdS_3-part is expressed with polar coordinates $(\phi, \gamma, \bar{\gamma})$

$$ds^2 = \ell^2(d\phi^2 + e^{2\phi} d\gamma d\bar{\gamma}),$$

where ℓ is the radius of the AdS_3 space and can be regarded as NS5 brane charges. This model also has non-vanishing NS-NS 2-form B_{NS} background

$$B_{NS} = \ell^2 e^{2\phi} d\gamma \wedge d\bar{\gamma}.$$

In the WZW model, we can incorporate this metric and the B_{NS} into a worldsheet lagrangean in the quantum corrected form. The bosonic part of the theory can be written with scalars ϕ, γ and auxiliary fields β, $\bar{\beta}$ with worldsheet spins 1

$$\mathcal{L} = \partial\phi\bar{\partial}\phi - \frac{2}{\alpha_+}R^{(2)}\phi + \beta\bar{\partial}\bar{\gamma} + \bar{\beta}\partial\bar{\gamma} - \beta\bar{\beta}\exp\left(-\frac{2\phi}{\alpha_+}\right),$$
$$\alpha_+ = \sqrt{2k-4}, \qquad (k = k_{s\ell} + 2).$$

This worldsheet theory has an affine $\mathrm{SL}(2;\mathbb{R})$ symmetry with level[1] $k = k_{s\ell}+2$ and its generators $\{J^a_B\}$ are expressed in the Wakimoto representation [13]

$$\begin{aligned} J^-_B &= \beta\,, \\ J^3_B &= \beta\gamma + \frac{\alpha_+}{2}\partial\phi\,, \\ J^+_B &= \beta\gamma^2 + \alpha_+\gamma\partial\phi + k\partial\gamma\,. \end{aligned}$$

Free fields $(\beta\,,\,\gamma)$ and ϕ have non-vanishing short distance behaviors only in the following cases

$$\beta(z)\gamma(w) \sim \frac{1}{z-w}\,, \qquad \phi(z)\phi(w) \sim -\log(z-w)\,.$$

Next we prepare a set of free fermions $\{\psi^a\}$ $(a=+,-,3)$ with OPEs

$$\begin{aligned} \psi^a(z)\psi^b(w) &\sim \frac{\eta^{ab}}{z-w}\,, \\ \eta^{+-} &= \eta^{-+} = 2\,, \qquad \eta^{33} = -1\,. \end{aligned}$$

These fields are combined into a set of affine $\mathrm{SL}(2;\mathbb{R})$ currents J^a_F represented as

$$\begin{aligned} J^a_F(z) &= -\frac{i}{2}\varepsilon^a{}_{bc}\psi^b\psi^c\,, \qquad (a=+,-,3)\,, \\ \varepsilon^{+-3} &= -2i\,. \end{aligned}$$

They constitute fermionic parts of an $\mathcal{N}=1$ affine $\mathrm{SL}(2;\mathbb{R})$ super algebra with a level $k_{s\ell}$. It is generated by a set of total currents $J^a = J^a_B + J^a_F$.

We apply the same recipe to describe two S^3 parts of the model by two affine $\mathrm{SU}(2)$ currents. Bosonic parts are generated by two sets of currents $K^{(I)a}$ $(I = 1,2\,;\, a=+,-,3)$

$$\begin{aligned} K^{(I)-}_B &= \beta^{(I)}\,, \\ K^{(I)3}_B &= \beta^{(I)}\gamma^{(I)} + i\sqrt{\frac{k'}{2}}\partial\phi^{(I)}\,, \\ K^{(I)+}_B &= \beta^{(I)}\gamma^{(I)2} - i\sqrt{2k'}\gamma^{(I)}\partial\phi^{(I)} + (k'-2)\partial\gamma^{(I)}\,, \\ \beta^{(I)}(z)\gamma^{(J)}(w) &\sim \frac{\delta^{IJ}}{z-w}\,, \qquad \phi^{(I)}(z)\phi^{(J)}(w) \sim -\delta^{IJ}\log(z-w)\,. \end{aligned}$$

In order to construct fermionic parts of the currents $K^{(I)a}_F$, we prepare two sets of fermions $\chi^{(I)a}$ $(I=1,2\,;\,a=+,-,3)$. One can combine them into $\mathcal{N}=1$ affine $\mathrm{SU}(2)$ currents with level 2

$$\begin{aligned} K^{(I)a}_F &= -\frac{i}{2}\varepsilon^a{}_{bc}\chi^{(I)b}\chi^{(I)c}\,, \\ \chi^{(I)a}(z)\chi^{(J)b}(w) &\sim \frac{\delta^{IJ}\tilde{\eta}^{ab}}{z-w}\,, \\ \tilde{\eta}^{+-} &= \tilde{\eta}^{-+} = 2\,, \qquad \tilde{\eta}^{33} = +1\,, \qquad \varepsilon^{+-3} = -2i\,. \end{aligned}$$

[1] The radius ℓ of AdS_3 is related to the level k of the $\mathrm{SL}(2;\mathbb{R})$ affine algebra $\ell^2 = k\ell_s^2$, $(\ell_s = \sqrt{\alpha'})$.

The total currents for each S^3 are defined as $K^{(I)a} = K_B^{(I)a} + K_F^{(I)a}$ $(I = 1,2\,;\, a = +,-,3)$ with a level k'. A set of a boson Y and a fermion λ describes a string propagating on S^1 and associated currents are given by ∂Y and λ.

Now let us consider an anomaly free condition, that is, a criticality condition of string theory. The matter parts in this system contain $\mathrm{SL}(2;\mathbb{R})_{k_{s\ell}}$, $\mathrm{SU}(2)_{k'}^{(I)}$ and U(1) affine Lie algebras and their centers must be balanced with those of ghost parts

$$15 = 3\left(\frac{3}{2} + \frac{2}{k_{s\ell}}\right) + 2\times 3\left(\frac{3}{2} - \frac{2}{k'}\right) + \frac{3}{2}\,.$$

It leads to a relation among two levels $k_{s\ell} = \dfrac{k'}{2}$. Then the worldsheet theory has a set of $\mathcal{N}=1$ superconformal currents $T(z)$ (energy momentum tensor) and $G(z)$ (super stress tensor)

$$\begin{aligned}
T(z) =\;& \frac{1}{k_{s\ell}}\left(\eta_{ab}J_B^a J_B^b + \frac{1}{2}\sum_{I=1,2}\tilde{\eta}_{ab}K_B^{(I)a}K_B^{(I)b}\right) - \\
& -\frac{1}{2}\left(\eta_{ab}\psi^a\partial\psi^b + \sum_{I=1,2}\tilde{\eta}_{ab}\chi^{(I)a}\partial\chi^{(I)b} + \partial Y\partial Y + \lambda\partial\lambda\right), \\
G(z) =\;& \sqrt{\frac{2}{k_{s\ell}}}\left(\eta_{ab}\psi^a J_B^b - \frac{i}{6}\varepsilon_{abc}\psi^a\psi^b\psi^c\right) + \\
& + \sqrt{\frac{1}{k_{s\ell}}}\sum_{I=1,2}\left(\tilde{\eta}_{ab}\chi^{(I)a}K_B^{(I)b} - \frac{i}{6}\varepsilon_{abc}\chi^{(I)a}\chi^{(I)b}\chi^{(I)c}\right) + i\lambda\partial Y\,.
\end{aligned} \qquad (2.3)$$

Now we shall explain the $\mathbb{Z}_2$-action of our model. In order to leave operators invariant under an exchange of two SU(2)'s, we set an action of the $\mathbb{Z}_2$ as

$$(K^{(1)}, K^{(2)}, Y) \to (K^{(2)}, K^{(1)}, -Y)\,,$$

where the $(K^{(1)}, K^{(2)}, Y)$ is a set of coordinates of $S^3\times S^3\times S^1$. This operation picks up only diagonal part of the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ algebra and an isometry "$\mathrm{SU}(2)\times\mathrm{SU}(2)$" of large $\mathcal{N}=4$ is reduced to a diagonal $\mathrm{SU}_D(2)$. This turns out to generate an $\mathcal{N}=3$ superconformal algebra.

First the diagonal $\mathrm{SU}_D(2)$ is invariant under the $\mathbb{Z}_2$ action and an associated affine Lie algebra is constructed by summing two SU(2)'s

$$K^a = K^{(1)a} + K^{(2)a}\,.$$

The worldsheet $\mathrm{SU}_D(2)$ generates an affine Lie algebra with a level $k_{so} = 2k' = 4k_{s\ell}$. That is to say, the diagonal "S^3" has a radius $\ell_s\sqrt{k_{so}} = 2\ell_s\sqrt{k_{s\ell}}$, which is twice as large as the radius $(\ell_s\sqrt{k_{s\ell}})$ of the AdS_3.

By using these currents, we can introduce mode operators of space-time affine SU(2) currents T^a_m in the 0-picture

$$T^a_m = \oint \frac{dz}{2\pi i}\left[\gamma^m K^a + 2m\gamma^m \chi^a\left(\psi^3 - \frac{1}{2}\psi^-\gamma - \frac{1}{2}\psi^+\gamma^{-1}\right)\right],$$
$$\psi^\pm = \psi^1 \pm i\psi^2\,, \qquad \chi^a = \frac{1}{\sqrt{2}}\left(\chi^{(1)a} + \chi^{(2)a}\right),$$

or in the (-1)-picture

$$T^a_m = \sqrt{2\,k_{s\ell}}\oint \frac{dz}{2\pi i} e^{-\varphi}\chi^a\gamma^m\,.$$

The φ is the bosonized super-reparametrization ghost. This space-time affine algebra has a level

$$\tilde{k}_{so} = 2k'p\,, \qquad p = \oint \frac{dz}{2\pi i}\gamma^{-1}\partial\gamma\,.$$

Also modes of a Virasoro operator L_m have the following formula in the 0-picture

$$L_m = -\oint \frac{dz}{2\pi i}\left[(1-m^2)J^3\gamma^m + \frac{m(m-1)}{2}J^-\gamma^{m+1} + \frac{m(m+1)}{2}J^+\gamma^{m-1}\right].$$

They satisfy space-time commutation relations with a central charge $\tilde{c} = 3k'p$ and two central terms are associated as $\tilde{c} = \frac{3}{2}\tilde{k}_{so}$.

Next the space-time fermionic generators are constructed from spin operators [17] in the worldsheet theory. We have ten free fermions in the associated super WZW model with the target space $AdS_3 \times S^3 \times S^3 \times S^1$:

$$\begin{cases} AdS_3 \cdots \psi^1\,, & \psi^2\,, \quad \psi^3\,, \\ S^3 \cdots \chi^{(1)1}\,, & \chi^{(1)2}\,, \quad \chi^{(1)3}\,, \\ S^3 \cdots \chi^{(2)1}\,, & \chi^{(2)2}\,, \quad \chi^{(2)3}\,, \\ S^1 \cdots \lambda\,. \end{cases}$$

Supercharges (spin operators) before the $\mathbb{Z}_2$-operation belong to the $(\mathbf{2}\,,\,\mathbf{2})$ representation of the $\mathrm{SU}(2)\times\mathrm{SU}(2)$. They are decomposed into $\mathbf{3}+\mathbf{1}$ representations under the $\mathrm{SU}_D(2)$. The $\mathbb{Z}_2$-action projects out the singlet and survives the triplet fermionic generators. In order to pick up $\mathbb{Z}_2$-invariant parts, we introduce next combinations

$$\begin{aligned} \boldsymbol{\psi}^1 &= \psi^1\,, & \boldsymbol{\psi}^2 &= \psi^2\,, \\ \boldsymbol{\psi}^3 &= \chi^{(1)1}\,, & \boldsymbol{\psi}^4 &= \chi^{(1)2}\,, \\ \boldsymbol{\psi}^5 &= \chi^{(2)1}\,, & \boldsymbol{\psi}^6 &= \chi^{(2)2}\,, \\ \boldsymbol{\psi}^7 &= \frac{1}{\sqrt{2}}\left(\chi^{(1)3} + \chi^{(2)3}\right), & \boldsymbol{\psi}^8 &= i\psi^3\,, \\ \boldsymbol{\psi}^9 &= \frac{1}{\sqrt{2}}\left(\chi^{(1)3} - \chi^{(2)3}\right), & \boldsymbol{\psi}^{10} &= \lambda\,, \end{aligned}$$

and will bosonize these fermions

$$\begin{aligned}
\boldsymbol{\psi}^1 \pm i\boldsymbol{\psi}^2 &= \sqrt{2}e^{\pm iH_1}\,, & \boldsymbol{\psi}^3 \pm i\boldsymbol{\psi}^4 &= \sqrt{2}e^{\pm iH_2}\,,\\
\boldsymbol{\psi}^5 \pm i\boldsymbol{\psi}^6 &= \sqrt{2}e^{\pm iH_3}\,, & \boldsymbol{\psi}^7 \pm i\boldsymbol{\psi}^8 &= \sqrt{2}e^{\pm iH_4}\,,\\
\boldsymbol{\psi}^9 \pm i\boldsymbol{\psi}^{10} &= \sqrt{2}e^{\pm iH_5}\,.
\end{aligned}$$

Then spin fields S_α are defined in the $(-1/2)$-picture as

$$\begin{aligned}
&S_\alpha = \exp\left(\frac{i}{2}\sum_I \alpha_I H^I\right) c_\alpha\,,\\
&c_\alpha\,;\ \text{cocycle factor}\,.
\end{aligned}$$

The α_I $(I = 1, 2, \ldots, 5)$ takes its value ± 1. These fields have non-trivial OPEs

$$\begin{aligned}
e^{-\varphi/2}S_\alpha(z)e^{-\varphi/2}S_\beta(w) &\sim \frac{1}{z-w}e^{-\varphi}(C\Gamma^M)_{\alpha\beta}\boldsymbol{\psi}^M\,,\\
\boldsymbol{\psi}^M(z)S_\alpha(w) &\sim \frac{1}{\sqrt{2}(z-w)^{1/2}}S_\beta(\Gamma^M)^\beta{}_\alpha\,,
\end{aligned} \tag{2.4}$$

with

$$\{\Gamma^M, \Gamma^N\} = 2\delta^{MN}\,, \qquad C\Gamma^M C^{-1} = -(\Gamma^M)^T\,, \qquad C^T = -C\,, \qquad C^2 = 1\,. \tag{2.5}$$

Here Γ^Ms are SO(10) gamma matrices and the C is a matrix representation of a charge conjugation operator. Also several concrete expressions are put in order in an appendix A. One can introduce a notation $u(z)$ to represent general types of spinors

$$u(z) = \sum_\alpha S_\alpha(z)u^\alpha\,.$$

We can express operator product expansions of the $u(z)$ and $\boldsymbol{\psi}^M$ as

$$\boldsymbol{\psi}^M(z)u(w) \sim \frac{1}{\sqrt{2}(z-w)^{1/2}}S_\beta(\Gamma^M)^\beta{}_\alpha u^\alpha\,.$$

As operations on the u^αs, the operators $\boldsymbol{\psi}^M$ can be identified with the $(1/\sqrt{2})\Gamma^M$.

Naively there seems to be possible 32 spin operators, but we have to impose several physical conditions on them. The first is the BRS condition and it acts on the $u(z)$. An associated part of this is described by an operator Γ_G

$$\begin{aligned}
&\Gamma_G = i\Gamma^1\Gamma^2\Gamma^8 + \frac{1}{2}\Gamma^3\Gamma^4(\Gamma^7+\Gamma^9) + \frac{1}{2}\Gamma^5\Gamma^6(\Gamma^7-\Gamma^9)\,,\\
&\Gamma_G u = 0\,.
\end{aligned} \tag{2.6}$$

The second condition comes from a GSO projection operator Γ^{11}

$$\begin{aligned}
\Gamma^{11} &= -i\Gamma^1\Gamma^2\cdots\Gamma^{10}\,,\\
\Gamma^{11}u &= +u\,.
\end{aligned} \tag{2.7}$$

One can impose a positive chirality condition on the left-part in the worldsheet $\Gamma^{11}u = +u$ only if the number of minus signs of α_I is even. Namely, only spinor representation is survived in this projection. Oppositely a negative chirality condition leads us to pick up some sets $\{\alpha_I\}$ with odd number of minus signs. It corresponds to a co-spinor representation.

The last constraint is reduced to the $\mathbb{Z}_2$ projection Γ_Ω. This $\mathbb{Z}_2$ operation acts on fermionic fields as

$$\begin{aligned}
(\Gamma^3\,,\,\Gamma^5) &\to (\Gamma^5\,,\,\Gamma^3)\,,\\
(\Gamma^4\,,\,\Gamma^6) &\to (\Gamma^6\,,\,\Gamma^4)\,,\\
(\Gamma^9\,,\,\Gamma^{10}) &\to (-\Gamma^9\,,\,-\Gamma^{10})\,,\\
\Gamma^M &\to \Gamma^M\,, \qquad (M = 1, 2, 7, 8)\,,
\end{aligned}$$

and is described by an operator Γ_Ω

$$\Gamma_\Omega = \frac{1}{2}\Gamma^9\Gamma^{10}(\Gamma^3+\Gamma^5)(\Gamma^4+\Gamma^6)\Gamma^3\Gamma^4\Gamma^5\Gamma^6\,, \qquad \text{with} \qquad \Gamma_\Omega^2 = 1\,.$$

$\mathbb{Z}_2$-invariant spinors must be selected out by a condition

$$\Gamma_\Omega u = +u\,. \tag{2.8}$$

Now we make a remark here: the operator Γ_Ω picks up a triplet in the $\mathrm{SU}_D(2)$. But we can equivalently project out a triplet but leave a singlet invariant by using another $\mathbb{Z}_2$-projection

$$\widetilde{\Gamma_\Omega} = -\Gamma_\Omega\,.$$

That leads us to obtain an $\mathcal{N}=1$ superconformal algebra as we will show in the next subsection.

We obtain physical spinor operators S_r^a $(a = 1, 2, 3, \,;\, r = \pm 1/2)$ satisfying the above three conditions (2.6), (2.7), (2.8)

$$\begin{aligned}
S^+_{+1/2} &= S_{+++++}\,,\\
S^0_{+1/2} &= -\frac{1}{2\sqrt{2}}(S_{+-+-+} + S_{+-++-} + S_{++--+} - S_{++-+-})\,,\\
S^-_{+1/2} &= -S_{+----}\,,\\
S^+_{-1/2} &= -S_{-++-+}\,,\\
S^0_{-1/2} &= \frac{1}{2\sqrt{2}}(S_{--+++} - S_{--+--} + S_{-+-++} + S_{-+---})\,,\\
S^-_{-1/2} &= -S_{---+-}\,.
\end{aligned}$$

The $\{S_r^a\}$s belong to representation $(\mathbf{2}\,,\,\mathbf{3})$ of the $\mathrm{SL}(2\,;\,\mathbb{R}) \times \mathrm{SU}_D(2)$ and the index "r" corresponds to a doublet of $\mathrm{SL}(2\,;\,\mathbb{R})$, the "$a$" is associated to a triplet in $\mathrm{SU}_D(2)$.

We can write down full modes of space-time super currents Q^a_r in the $(-1/2)$-picture

$$
\begin{aligned}
Q^a_r &= A \oint \frac{dz}{2\pi i} e^{-\varphi/2} S^a_r \,, \\
S^a_r &= \left(r+\frac{1}{2}\right)\gamma^{r-1/2} S^a_{+1/2} - \left(r-\frac{1}{2}\right)\gamma^{r+1/2} S^a_{-1/2} \,, \\
A^2 &= -i(8\,k_{s\ell})^{1/2} \,, \qquad \left(a=1,2,3\,;\, r\in\frac{1}{2}+\mathbb{Z}\right) .
\end{aligned}
$$

These currents L_n, T^a_m, Q^a_r close commutation relations together with a spin 1/2 fermionic current Ψ_r

$$
\begin{aligned}
\Psi_r &= A \oint \frac{dz}{2\pi i} e^{-\varphi/2} P_r \,, \\
P_r &= -\frac{1}{2\sqrt{2}}\Big[\gamma^{r+1/2}(-S_{--+++} - S_{--+--} - S_{-+-++} + S_{-+---}) + \\
&\qquad + \gamma^{r-1/2}(-S_{+-+-+} + S_{+-++-} - S_{++--+} - S_{++-+-})\Big] .
\end{aligned}
$$

These constitute a space-time $\mathcal{N}=3$ SCA [18, 19, 20] with central charge $\tilde{c}=\frac{3}{2}\tilde{k}_{so}$, $(k_{so}=4\,k_{s\ell}\,,\ \tilde{k}_{so}=k_{so}p)$

$$
\begin{aligned}
[L_m, L_n] &= (m-n)L_{m+n} + \frac{\tilde{c}}{12}(m^3-m)\delta_{m+n,0} \,, \\
[T^a_m, T^b_n] &= i\varepsilon^{ab}{}_c T^c_{m+n} + \frac{\tilde{k}_{so}}{2} m\delta_{m+n,0}\tilde{\eta}^{ab} \,, \\
[L_m, T^a_n] &= -nT^a_{m+n} \,, \\
[T^a_m, Q^b_r] &= i\varepsilon^{ab}{}_c Q^c_{m+r} + \tilde{\eta}^{ab} m \Psi_{m+r} \,, \\
[L_m, Q^b_r] &= \left(\frac{1}{2}m - r\right) Q^b_{m+r} \,, \\
\{Q^a_r, Q^b_s\} &= 2\tilde{\eta}^{ab} L_{r+s} + i\varepsilon^{ab}{}_c (r-s) T^c_{r+s} + \frac{1}{3}\tilde{c}\left(r^2-\frac{1}{4}\right)\delta_{r+s,0}\tilde{\eta}^{ab} \,, \\
[L_m, \Psi_r] &= \left(-\frac{1}{2}m - r\right)\Psi_{m+r} \,, \qquad [T^a_m, \Psi_r] = 0 \,, \\
\{Q^a_r, \Psi_s\} &= T^a_{r+s} \,, \qquad \{\Psi_r, \Psi_s\} = \frac{\tilde{k}_{so}}{2}\delta_{r+s,0} \,.
\end{aligned}
$$

The $\mathcal{N}=3$ superconformal algebra has a global subalgebra OSp(3|2) whose bosonic part is associated with an isometry of the $AdS_3 \times S^3$. Also super Lie algebra OSp(3|2) is an O(3) extended anti-de Sitter group in two dimension. Modes of super stress tensors in space-time belong to vector representation **3** of this O(3) $\subset$ OSp(3|2). It is contrasted with the fact that the super stress tensors in the (small or large) $\mathcal{N}=4$ SCFT belong to spinor representation of the SU(2) $\subset$ SU$(1,1|2)$ or SU$(1,1|2) \times$ SU$(1,1|2)$ respectively.

Let us comment on space-time physical operators. We construct vertex operators associated to SL$(2;\mathbb{R})\times$SU$(2)^{(1)}\times$SU$(2)^{(2)}$. We introduce following operators

$$
\begin{aligned}
V_{j,m} &= \gamma^{j+m} \exp\left[\frac{2j\phi}{\alpha_+}\right], \\
W^{(1)}_{j,m} &= \frac{1}{\sqrt{(j+m)!(j-m)!}}\gamma^{(1)j+m} \exp\left[-i\frac{2j\phi^{(1)}}{\alpha'_+}\right], \\
W^{(2)}_{j,m} &= \frac{1}{\sqrt{(j+m)!(j-m)!}}\gamma^{(2)j+m} \exp\left[-i\frac{2j\phi^{(2)}}{\alpha'_+}\right], \\
\alpha_+ &= \sqrt{2k_{s\ell}}, \qquad \alpha'_+ = \sqrt{2k'}\,.
\end{aligned}
$$

The $V_{j,m}$ is the vertex operator in the SL$(2;\mathbb{R})$ algebra and each $W^{(I)}_{j,m}$ $(I=1,2)$ belongs to a $K_0^{(I)3} = m$ state in the spin j representation of the affine SU$(2)^{(I)}$ algebra. Then we present a vertex operator of the space-time SCFT in the (-1)-picture

$$
V = e^{-\varphi}(\psi^-\gamma + \psi^+\gamma^{-1} - 2\psi^3)V_{j,m}W^{(1)}_{j',m'}W^{(2)}_{j'',m''}\,.
$$

Its worldsheet conformal dimension h is calculated as

$$
h = \frac{1}{2} + \frac{1}{2} - \frac{1}{k_{s\ell}}j(j+1) + \frac{1}{k'}j'(j'+1) + \frac{1}{k'}j''(j''+1)\,.
$$

As a physical condition on the operator V, this h should be 1 and we obtain a relation

$$
\frac{1}{k_{s\ell}}j(j+1) = \frac{1}{k'}j'(j'+1) + \frac{1}{k'}j''(j''+1)\,.
$$

When one uses the level relation $2k_{s\ell} = k'$ (criticality condition), it becomes an equation for the set (j, j', j'')

$$
j(j+1) = \frac{1}{2}\left[\, j'(j'+1) + j''(j''+1)\,\right].
$$

We have to impose a $\mathbb{Z}_2$-invariant condition for the operators. In this paper we restrict ourselves to the $j' = j''$ case, for simplicity. In this case, a relation $j = j'$ $(= j'')$ is automatically satisfied, and the $\mathbb{Z}_2$-invariant operators turn out to be the spin $2j$ representation of the SU$_D(2)$. Moreover, the highest weight state of this multiplet corresponds to a space-time primary state with space-time conformal dimension $\tilde{h} = j$ and space-time SU(2) spin $\tilde{j} = 2j$. This state with $2\tilde{h} = \tilde{j}$ represents a chiral primary field of the space-time $\mathcal{N} = 3$ theory.

Now we recall that the short multiplet of the $\mathcal{N} = 3$ SCFT is constructed by multiplying $Q^-_{-1/2}, Q^0_{-1/2}$ to the chiral primary state $|\tilde{h}, \tilde{j} = 2\tilde{h}\rangle$ as shown in figure 1. Let us consider these space-time chiral primaries. First we perform a standard twisting method on the space-time $\mathcal{N} = 3$ superconformal algebra. All we have to do is

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$$
\begin{tikzcd}
 & T_0^0 = 2\tilde{h} & T_0^0 = 2\tilde{h} - 1 \\
L_0 = \tilde{h} & \boxed{|\tilde{h}, \tilde{j} = 2\tilde{h}\rangle} \arrow[d] \arrow[dr] & \\
L_0 = \tilde{h} + \frac{1}{2} & \boxed{Q^0_{-1/2}|\tilde{h}, 2\tilde{h}\rangle} \arrow[r, leftrightarrow] \arrow[dr] & \boxed{Q^-_{-1/2}|\tilde{h}, 2\tilde{h}\rangle} \arrow[d] \\
L_0 = \tilde{h} + 1 & & \boxed{Q^-_{-1/2} Q^0_{-1/2}|\tilde{h}, 2\tilde{h}\rangle}
\end{tikzcd}
$$

Figure 1: Short multiplet of the $\mathcal{N} = 3$ SCA. The state $|\tilde{h}, \tilde{j} = 2\tilde{h}\rangle$ is the chiral primary state. Other states in the multiplet are derived by multiplying $Q^-_{-1/2}$, $Q^0_{-1/2}$ on the chiral primary.

to modify modes $(\tilde{L}_m, \tilde{Q}^0_r)$ of the energy momentum tensor and its ($\mathrm{SU}_D(2)$ neutral) super partner as

$$
\begin{aligned}
\tilde{L}_m &= L_m - \frac{1}{2}(m+1) T^0_m \,, \\
\tilde{Q}^0_r &= Q^0_r - \left(r + \frac{1}{2}\right) \Psi_r \,, \\
\tilde{L}_0 &= L_0 - \frac{1}{2} T^0_0 \,.
\end{aligned}
\tag{2.9}
$$

This operation changes spins of charged currents under SU(2) and the original $\mathcal{N} = 3$ contents are rearranged into four sets of fields in table 1. The spin one current $Q^+_{r=-1/2}$ plays a role of a BRST operator Q_{BRS}

$$
Q_{BRS} = Q^+_{-1/2} \,,
$$

and the modes of the energy momentum tensor are expressed in the BRST exact form

$$
\tilde{L}_n = \frac{1}{4} \left\{ Q_{BRS}, Q^-_{n+\frac{1}{2}} \right\} \,.
$$

Similarly the currents $\tilde{Q}^0_r$, Q^+_r, T^+_m are respectively written as BRST exact forms of the T^-_m, T^0_m, Ψ_r. Under this modification, only short multiplets are picked up as physical states automatically because states $Q^+_{-1/2}|\cdots\rangle = Q_{BRS}|\cdots\rangle$ are unphysical in that case.

Also two components of four sets $(\tilde{L}_m, \tilde{Q}^0_r)$, (Q^-_r, T^-_m), (Q^+_r, T^+_m), (T^0_m, Ψ_r) are connected each other by the super charge Q_s with spin 1/2

$$
Q_s = Q^0_{-1/2} \,.
$$

In particular, $\{Q_s, \tilde{Q}^0_{n+\frac{1}{2}}\} = 2\tilde{L}_n$ is satisfied. This topological model has an $\mathcal{N}=1$ supersymmetry and the states $|\tilde{h}, 2\tilde{h}\rangle$, $Q^0_{-1/2}|\tilde{h}, 2\tilde{h}\rangle$ in the short multiplet are understood as a doublet of this susy algebra. From the eq. (2.9), we observe that an arbitrary state with $(L_0, T^0_0) = (\tilde{h}, \tilde{j})$ is relabeled by a set of new eigenvalues of $(\tilde{L}_0, T^0_0)$ as $(\tilde{h} - \frac{1}{2}\tilde{j}, \tilde{j})$. In particular, original chiral primary state with $2\tilde{h} = \tilde{j}$ has eigenvalue zero for $\tilde{L}_0$ and is specified by the eigenvalue $\tilde{j}$ of T^0_0. Also the state $Q^-_{-1/2}|\tilde{h}, 2\tilde{h}\rangle$ has dimension 1 under the $\tilde{L}_0$ (with charge $2\tilde{h}-1$) and integrated form of an associated scaling operator is interpreted as a marginal operator in space-time theory. We may call the fields as moduli of some associated sigma model.

We propose these sets of fields as candidates of physical operators in some sigma model. Its target space might be a symmetric product (Hilbert scheme) of some manifold. We would like to discuss these subjects in future work.

Spin	2	3/2	1	1/2
Fields	$\tilde{L}_m$	$\tilde{Q}^0_r$	Q^+_r	T^+_m
Fields	Q^-_r	T^-_m	T^0_m	Ψ_r

Table 1: Spins of the twisted $\mathcal{N}=3$ theory. When one modifies the L_m, Q^0_r into the $\tilde{L}_m$, $\tilde{Q}^0_r$, the spins of the charged currents are changed by $\pm 1/2$.

Next we study this model in terms of worldsheet CFT and make some speculations. The zero-mode part of the SU(2) current is constructed by the two $\mathcal{N}=1$ affine SU(2) currents in the worldsheet

$$\begin{aligned} T^0_0 &= \oint \frac{dz}{2\pi i} K^3 \,, \qquad K^3 = K^{(1)3} + K^{(2)3} \,, \\ K^{(I)3} &= \beta^{(I)}\gamma^{(I)} + i\sqrt{k_{s\ell}}\,\partial\phi^{(I)} + \frac{1}{2}\chi^{(I)+}\chi^{(I)-} \,, \qquad (I=1,2) \,, \end{aligned} \tag{2.10}$$

and the zero-mode of the modified stress tensor is expressed by the SL(2; ℝ) current together with K^3

$$\tilde{L}_0 = \oint \frac{dz}{2\pi i}\left(-J^3 - \frac{1}{2}K^3\right), \qquad J^3 = \beta\gamma + i\sqrt{\frac{k_{s\ell}}{2}}\,\partial\phi + \frac{1}{2}\psi^+\psi^- \,. \tag{2.11}$$

The currents (2.10), (2.11) specify one complex structure of the (product of) manifolds $AdS_3 \times S^3_D \cong \mathrm{SL}(2;\mathbb{R}) \times \mathrm{SU}_D(2)$ and the model is reduced to a coset type space

$$\frac{\mathrm{SL}(2;\mathbb{R})_{k_{s\ell}}}{\mathrm{U}(1)} \times \frac{\mathrm{SU}_D(2)_{k_{so}}}{\mathrm{U}(1)} \times \mathrm{U}(1)^2 \,.$$

In terms of the worldsheet CFT, the currents J^3, K^3 are parts of some U(1) current J. Also, it is well-known that the $N=1$ superconformal algebra (2.3) is enhanced to an $N=2$ CFT generated by T, $G = G^+ + G^-$ and J in this coset case. Now we concentrate on the $\mathrm{SU}_D(2)_{k_{so}}/\mathrm{U}(1)$ part. It is a special case of the Grassmannian model

$$\frac{\mathrm{SU}(n+m)_k}{\mathrm{SU}(n)\otimes\mathrm{SU}(m)\otimes\mathrm{U}(1)} \qquad \text{with} \qquad n=m=1\,, k=k_{so}\,.$$

When we use the level-rank duality of the coset model, this case is reduced to a level 1 model

$$\frac{\mathrm{SU}(2)_{k_{so}}}{\mathrm{U}(1)} \rightarrow \frac{\mathrm{SU}(k_{so}+1)_1}{\mathrm{U}(k_{so})} \, .$$

It is nothing but a $CP^{k_{so}}$-model with central charge $c = 3\,(1 - 2/k_{so})$ and is described by a $A_{k_{so}+1}$ type minimal series in the worldsheet $N = 2$ theory or 2dim Landau-Ginzburg theory. There could be a correspondence between operators of this CP-model and those of AdS_3 model.

Now we make several remarks here: the level $k_{s\ell}$ can be interpreted as the square of the radius of the AdS_3. In our case, it is related with the $k_{so} = 4k_{s\ell}$ of the SU(2) level and k_{so} must be integer. That is to say, the associated CFTs are realized only on isolated integer points on the k_{so} line. When one fixes the k_{so} as some number, the associated CFT has at most finite number of primary fields representing degenerate vacua of Landau-Ginzburg model. However we can formally resolve the degeneracy of the vacua by perturbation while preserving $N = 2$ symmetry. It is an interesting problem to clarify connections between the interpolating theory connecting different vacua and AdS_3 geometry.

2.3 $\mathcal{N} = 1$ affine super Lie algebra

Until now, we concentrate on the left-moving part of the worldsheet theory. It generates a space-time $\mathcal{N} = 3$ SCA in the left-handed side. In order to construct full theory, we have to consider the remaining right-moving anti-holomorphic part. The same recipes[2] can be applied to this case in the bosonic parts. Also one imposes conditions on right-moving spinors $\bar{u}(\bar{z})$

$$\Gamma_G \bar{u} = 0 \, , \qquad \Gamma_\Omega \bar{u} = +\bar{u} \, ,$$

as physical conditions. But we have to impose a negative chirality condition (GSO projection) on the $\bar{u}$

$$\Gamma^{11} \bar{u} = -\bar{u} \, ,$$

GSO $\diagdown$ $\mathbb{Z}_2$	$+$	$-$
$+$	$\mathcal{N} = 3$	$\mathcal{N} = 1$
$-$	$\mathcal{N} = 1$	$\mathcal{N} = 3$

Table 2: Choices of the GSO and $\mathbb{Z}_2$ projections and space-time superconformal symmetries. The signatures $(+)$, $(-)$ for the GSO and $\mathbb{Z}_2$ projections mean that the corresponding eigenvalues for these operators Γ^{11}, Γ_Ω are $+1$, -1 respectively.

because we consider the type IIA string. We summarize possible choices of the GSO and $\mathbb{Z}_2$ projections and associated space-time supersymmetries in table 2.

Here, we take a choice that the GSO projection is "$-$" and $\mathbb{Z}_2$ projection is "$+$" case. Then we get the $\mathcal{N} = 1$ SCA in the right-handed side. In fact these conditions

[2] We use the symbol "$^-$" (bar) to distinguish right-handed fields from the associated left-handed ones.

select out only two possible spin operators $\bar{S}_{\pm 1/2}$

$$\bar{S}_{+1/2} = \frac{1}{2\sqrt{2}}(+\bar{S}_{+-+++} + \bar{S}_{+-+--} + \bar{S}_{++-++} - \bar{S}_{++---}) ,$$
$$\bar{S}_{-1/2} = \frac{1}{2\sqrt{2}}(+\bar{S}_{--+-+} - \bar{S}_{--++-} + \bar{S}_{-+--+} + \bar{S}_{-+-+-}) .$$

They belong to a representation $(\mathbf{1},\mathbf{2})$ of $\mathrm{SU}_D(2)\times\mathrm{SL}(2;\mathbb{R})$. In this case, a triplet of $\mathrm{SU}_D(2)$ is projected out and only $\mathcal{N}=1$ algebra is retained. Then we can construct a set of modes of associated supercharge $\bar{Q}_r$

$$\bar{Q}_r = A\oint \frac{d\bar{z}}{2\pi i} e^{-\bar{\varphi}/2}\left[\left(r+\frac{1}{2}\right)\bar{\gamma}^{r-1/2}\bar{S}_{+1/2} - \left(r-\frac{1}{2}\right)\bar{\gamma}^{r+1/2}\bar{S}_{-1/2}\right],$$

in the space-time right-handed side. They satisfy an $\mathcal{N}=1$ superconformal algebra

$$\begin{aligned}
[\bar{L}_m,\bar{L}_n] &= (m-n)\bar{L}_{m+n} + \frac{\tilde{c}}{12}(m^3-m)\delta_{m+n,0} ,\\
[\bar{L}_m,\bar{Q}_r] &= \left(\frac{1}{2}m - r\right)\bar{Q}_{m+r} ,\\
\{\bar{Q}_r,\bar{Q}_s\} &= 2\bar{L}_{r+s} + \frac{\tilde{c}}{3}\left(r^2-\frac{1}{4}\right)\delta_{r+s,0} ,
\end{aligned}$$

combined with Virasoro generators $\bar{L}_m$ in the right-moving side. But the story is not completed yet. We have a degree of freedom of the diagonal S^3, that is, the space-time $\mathrm{SU}_D(2)$ affine symmetry. We obtain modes of a set of the $\mathrm{SU}_D(2)$ generators $(\bar{T}^a_m,\bar{\Psi}^a_r)$ with $(a=+,-,3\,;\, r,s\in\frac{1}{2}+\mathbb{Z}\,;\, m\in\mathbb{Z})$

$$\begin{aligned}
\bar{T}^a_m &= \oint\frac{d\bar{z}}{2\pi i}\left[\bar{\gamma}^m\bar{K}^a + 2m\bar{\gamma}^m\bar{\chi}^a\left(\bar{\psi}^3 - \frac{1}{2}\bar{\psi}^-\bar{\gamma} - \frac{1}{2}\bar{\psi}^+\bar{\gamma}^{-1}\right)\right] \qquad (0\text{-picture}) ,\\
\bar{\Psi}^a_r &= A\oint\frac{d\bar{z}}{2\pi i}e^{-\bar{\varphi}/2}\bar{P}^a_r \qquad ((-1/2)\text{-picture}) ,\\
\bar{P}^+_r &= +\bar{\gamma}^{r+1/2}\bar{S}_{-++++} - \bar{\gamma}^{r-1/2}\bar{S}_{+++-+} ,\\
\bar{P}^0_r &= +\bar{\gamma}^{r+1/2}\left(-\bar{S}_{--+-+} - \bar{S}_{--++-} - \bar{S}_{-+--+} + \bar{S}_{-+-+-}\right) +\\
&\quad +\bar{\gamma}^{r-1/2}\left(+\bar{S}_{+-+++} - \bar{S}_{+-+--} + \bar{S}_{++-++} + \bar{S}_{++---}\right) ,\\
\bar{P}^-_r &= -\bar{\gamma}^{r+1/2}\bar{S}_{-----} - \bar{\gamma}^{r-1/2}\bar{S}_{+--+-} .
\end{aligned}$$

Thus space-time affine symmetry is a semi-direct product of the $\mathcal{N}=1$ super Virasoro and the $\mathcal{N}=1$ affine SU(2) algebra. They are generated by right-moving currents satisfying commutation relations

$$\begin{aligned}
[\bar{T}^a_m,\bar{T}^b_n] &= i\varepsilon^{ab}{}_c\bar{T}^c_{m+n} + \frac{\tilde{k}_{so}}{2}m\tilde{\eta}^{ab}\delta_{m+n,0} ,\\
[\bar{T}^a_m,\bar{\Psi}^b_r] &= i\varepsilon^{ab}{}_c\bar{\Psi}^c_{m+r} , &\qquad \{\bar{\Psi}^a_r,\bar{\Psi}^b_s\} &= \frac{\tilde{k}_{so}}{2}\tilde{\eta}^{ab}\delta_{r+s,0} ,\\
[\bar{Q}_r,\bar{T}^a_m] &= -m\bar{\Psi}^a_{m+r} , &\qquad \{\bar{Q}_r,\bar{\Psi}^a_s\} &= \bar{T}^a_{r+s} ,\\
[\bar{L}_m,\bar{T}^a_n] &= -n\bar{T}^a_{m+n} , &\qquad [\bar{L}_m,\bar{\Psi}^a_r] &= \left(-\frac{1}{2}m - r\right)\bar{\Psi}^a_{m+r} .
\end{aligned}$$

We collect these results in table 3.

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	IIA Theory	
space-time	left-side	right-side
chirality	$+$	$-$
symmetry	($\mathcal{N}=3$ Virasoro)	($\mathcal{N}=1$ Virasoro) $\ltimes$ ($\mathcal{N}=1$ affine SU(2))
generators	$L_m,\ Q_r^a,\ T_m^a,\ \Psi_r$	$\bar{L}_m,\ \bar{Q}_r,\ \bar{T}_m^a,\ \bar{\Psi}_r^a$

Table 3: The structure of the space-time CFT. The chiral projections act on left-, right-sides with opposite signatures in the IIA string. It leads to generate a left-right asymmetric $(\mathcal{N},\overline{\mathcal{N}})=(3,1)$ superconformal algebra.

3. The brane configuration

In this section, we study a brane configuration realized by M2-,M5-branes with a $\mathbb{Z}_2$-fixed plane in M-theory and show that it has a near horizon geometry $AdS_3 \times (S^3 \times S^3 \times \mathbb{R})/\mathbb{Z}_2 \times \mathbb{R}$. If the last real line $\mathbb{R}$ is compactified to a small circle S^1, the geometry becomes $AdS_3 \times (S^3 \times S^3 \times \mathbb{R})/\mathbb{Z}_2$ in the IIA theory considered in the previous section.

In the M5-M2 system, each brane extends to space-time directions shown in table 4. The fixed plane extends to oblique directions. For convenience to express this fixed plane, we change coordinates from x to x'

$$
\begin{aligned}
x'^p &= x^p, \quad (p=0,1,10),\\
x'^m &= \frac{1}{\sqrt{2}}(x^m + x^{m+4}), \quad (m=2,3,4,5),\\
x'^{m'} &= \frac{1}{\sqrt{2}}(x^{m'-4} - x^{m'}), \quad (m'=6,7,8,9).
\end{aligned}
$$

Then extended directions of the M5s and the M2s are retained as in table 4 in old coordinates x. But the fixed plane is stretched in directions 0, 1, 2, 3, 4, 5, 10, in the new coordinates x'.

direction	0	1	2	3	4	5	6	7	8	9	10
$\mathrm{M5}^{(1)}$	○	○					○	○	○	○	
$\mathrm{M5}^{(2)}$	○	○	○	○	○	○					
fixed plane	○	○	/	/	/	/	/	/	/	/	○
M2	○	○									○

Table 4: M5-M2 system. Each brane extends to directions marked by circles. The symbol "/" (slash) means that the fixed plane extends to the oblique directions. The $\mathrm{M5}^{(1)}$ and the $\mathrm{M5}^{(2)}$ are not independent sets of branes, but one is the mirror image of the other.

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When we compactify the x^{10} direction to a small S^1, we get a configuration with NS5-NS1 branes in the IIA theory.

Now we will consider a classical solution associated with this brane system. First we set the number of M5-branes $M5^{(1)}$ ($\sharp M5^{(1)}$) to be equal to that of $M5^{(2)}$s ($\sharp M5^{(2)}$). This set-up makes a $\mathbb{Z}_2$ action possible, which exchanges $M5^{(1)}$s and $M5^{(2)}$s one another.

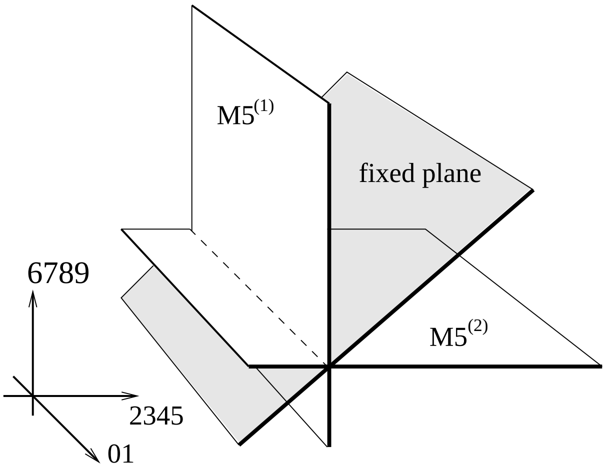

Figure 2: M5-M2 system. The M2 is omitted. The $M5^{(2)}$ is the mirror image of the $M5^{(1)}$ with respect to the fixed plane.

There is a metric of the classical solution in the $M5^{(1)}$-$M5^{(2)}$-M2 system [21] with $\sharp M5^{(1)} = \sharp M5^{(2)}$

$$
\begin{aligned}
ds^2 = {} & H_T^{1/3} H_{F1}^{2/3} H_{F2}^{2/3} \Big[(H_T H_{F1} H_{F2})^{-1} \Big\{ -(dx^0)^2 + (dx^1)^2 \Big\} + \\
& + H_{F1}^{-1} \Big\{ (dx^2)^2 + \cdots + (dx^5)^2 \Big\} + H_{F2}^{-1} \Big\{ (dx^6)^2 + \cdots + (dx^9)^2 \Big\} + H_T^{-1} (dx^{10})^2 \Big] , \\
H_{F2} = {} & 1 + \frac{R_5^2}{r_1^2} , \qquad H_{F1} = 1 + \frac{R_5^2}{r_2^2} , \qquad H_T = 1 + \frac{R_2^4}{r_1^2 r_2^2} , \\
r_1^2 = {} & (x^2)^2 + (x^3)^2 + (x^4)^2 + (x^5)^2 , \qquad r_2^2 = (x^6)^2 + (x^7)^2 + (x^8)^2 + (x^9)^2 .
\end{aligned}
$$

The R_5^2, R_2^4 are respectively constants proportional to the numbers of M5-branes Q_5, M2-branes Q_2. This metric is invariant under following $\mathbb{Z}_2$ action

$$
x^m \to x^{m+4} , \qquad x^{m+4} \to x^m , \qquad (m = 2, 3, 4, 5) .
$$

So we can divide this space by the $\mathbb{Z}_2$ and construct a classical M5-M2 solution with a $\mathbb{Z}_2$-fixed plane.

Now let us take a near horizon limit and see what the geometry is. The limit magnifies regions in the ranges $r_1, r_2 << R_2$ and $r_1, r_2 << R_5$ and then the above metric becomes

$$
\begin{aligned}
ds^2 = {} & \left(\frac{R^4}{r_1^2 r_2^2} \right)^{-1} \{ -(dx^0)^2 + (dx^1)^2 \} + \\
& + \frac{R^2}{r_1^2} dr_1^2 + R^2 \left(d\Omega_3^{(1)} \right)^2 + \frac{R^2}{r_2^2} dr_2^2 + R^2 \left(d\Omega_3^{(2)} \right)^2 + \left(\frac{R_5^4}{R_2^4} \right)^{2/3} \left(dx^{10} \right)^2 .
\end{aligned}
$$

Here the $(d\Omega_3^{(1)})^2$ and $(d\Omega_3^{(2)})^2$ are the metrics of two unit S^3s, and R is defined to be $(R_2^2 R_5)^{1/3}$ proportional to $(Q_2 Q_5)^{1/6}$. This metric is rather complicated and it is difficult to study the geometry directly. To make the geometrical structure more clearly, we perform a coordinate transformation $(r_1, r_2) \to (u, v)$ by

$$u = \ell \frac{r_1 r_2}{R^2}\,, \qquad v = \ell \log\left(\frac{r_1}{r_2}\right)\,, \qquad 2\ell^2 = R^2.$$

Then we get a near horizon metric of this system,

$$\begin{aligned} ds^2 = & \frac{\ell^2}{u^2}\Big\{du^2 - (dx^0)^2 + (dx^1)^2\Big\} + R^2\Big\{(d\Omega_3^{(1)})^2 + (d\Omega_3^{(2)})^2\Big\} + \\ & + dv^2 + \left(\frac{R_5^4}{R_2^4}\right)^{2/3} (dx^{10})^2\,. \end{aligned}$$

We find that the geometry is $AdS_3 \times S^{3\,(1)} \times S^{3\,(2)} \times \mathbb{R}^2$ before the $\mathbb{Z}_2$ operation.

When we treat M-theory as 11-dimensional classical supergravity, we need to take the limit $R \gg \ell_p$, where ℓ_p is the 11-dimensional Planck length. This is equivalent to a "large N" limit,

$$Q_2 Q_5 \gg 1\,.$$

Also, in our solution, both radii of the two S^3s are R. The parameter ℓ is identified with a radius of the AdS_3. They are connected by a relation

$$2\ell^2 = R^2\,. \tag{3.1}$$

It relates a level of the worldsheet affine algebra $\widehat{\mathrm{SU}(2)}_{k'}$ with that of the $\widehat{\mathrm{SL}(2,\mathbb{R})}_{k_{s\ell}}$

$$2k_{s\ell} = k'\,.$$

Now let us return to the $\mathbb{Z}_2$ action. If we let $\Omega^{(1)a}$ and $\Omega^{(2)a}$, $(a = 1, 2, 3)$ to be coordinates of $S^{3\,(1)}$ and $S^{3\,(2)}$ respectively, the $\mathbb{Z}_2$ operation acts on them

$$\Omega^{(1)a} \to \Omega^{(2)a}\,, \qquad \Omega^{(2)a} \to \Omega^{(1)a}\,, \qquad v \to -v\,.$$

Here we find that the $\mathbb{Z}_2$ action considered in the previous section is a natural operation. After the division, we obtain a resulting space $AdS_3 \times (S^3 \times S^3 \times \mathbb{R})/\mathbb{Z}_2 \times \mathbb{R}$. Because the x^{10} direction is flat in this geometry, we can compactify it to a small circle S^1. By decreasing the radius of the S^1, the geometry in the M-theory is reduced to the $AdS_3 \times (S^3 \times S^3 \times \mathbb{R})/\mathbb{Z}_2$ in the type IIA theory. It is just the same theory that we consider in the previous section.

Finally we comment on the fixed plane. On this fixed surface, next relations are satisfied

$$\begin{aligned} \Omega^{(1)a} &= \Omega^{(2)a} = \Omega_3^a\,, \qquad & v &= 0\,, \\ d\Omega^{(1)a} &= d\Omega^{(2)a} = d\Omega_3^a\,, \qquad & dv &= 0\,. \end{aligned}$$

The geometry of this surface is characterized as $AdS_3 \times S^3 \times \mathbb{R}$ and its induced metric is given as

$$ds^2 = \frac{\ell^2}{u^2}\Big\{du^2 - (dx^0)^2 + (dx^1)^2\Big\} + 2R^2(d\Omega_3)^2 + \left(\frac{R_5^4}{R_2^4}\right)^{2/3}(dx^{10})^2 \,.$$

A radius of S^3 part is understood to be $\tilde{R} = \sqrt{2}R$. By comparing the radius ℓ of AdS_3 with this $\tilde{R}$, we obtain a relation between them. They are connected with an equation

$$4\ell^2 = \tilde{R}^2 \,.$$

It leads to give us a ratio of two levels of the worldsheet $\widehat{\mathrm{SL}(2,\mathbb{R})}_{k_{s\ell}}$ and the diagonal $\widehat{\mathrm{SU_D}(2)}_{k_{so}}$ as

$$\frac{k_{so}}{k_{s\ell}} = \frac{\tilde{R}^2}{\ell^2} = 4 \,.$$

It coincides with the result derived in the previous section.

In this brane configuration, the M2-brane ends on the M5s. The intersecting part is a (1+1)-dimensional object and represents a string. When one compactifies one longitudinal dimension, it is known as a little string [22] trapped in the NS5-brane in the context of the small IIA theory. The associated low energy theory is believed to be an exotic 6dim (2,0) type IIA susy theory. In our set-up, our susy model lives on a fixed plane with these M2-M5 backgrounds. We might have a new low energy susy theory with some additional multiplets. We want to study these and clarify a description of them in the M(atrix) or Quiver matrix theory in future works.

4. Conclusions and discussions

In this paper, we investigated the superstring on the background $AdS_3 \times (S^3 \times S^3 \times S^1)/\mathbb{Z}_2$ in the framework presented in [2] and developed a method to construct a conformal theory with less supersymmetry in space-time. We study a WZW model on AdS_3 target space with a $\mathbb{Z}_2$-action and find that an $\mathrm{SL}(2;\mathbb{R}) \times \mathrm{SU}_D(2)$ isometry is enhanced to the $\mathcal{N} = 3$ superconformal algebra in space-time. Global symmetry part is a super Lie algebra OSp(3|2) which is an O(3) extended anti-de Sitter group in two dimension.

The criticality condition in this superstring is equivalent to a relation $k_{so} = 4k_{s\ell}$ of two levels k_{so}, $k_{s\ell}$ associated with affine Lie algebras $\mathrm{SU}_D(2)$ and $\mathrm{SL}(2;\mathbb{R})$ respectively. It also gives us information that a ratio among the radii of the diagonal S^3_D and AdS_3 is two.

In order to construct supercharges (spin operators), we used standard bosonization techniques for worldsheet fermions. We imposed three constraints on these spin operators as physical conditions. The first is the usual BRS condition. The second is reduced to the $\mathbb{Z}_2$-invariance condition. The $\mathbb{Z}_2$-operation acts on two S^3's and exchanges their associated fields one another. Under the $\mathbb{Z}_2$-projection, a diagonal part of products of two S^3's is survived and it serves as an R-symmetry of the space-time theory. It also changes signatures of fields on the circle S^1. The last condition comes from a GSO projection.

When one imposes a positive chirality condition, only spinor representation is survived. Oppositely co-spinor operators are obtained under a negative chirality constraint. We have four possible choices for these $\mathbb{Z}_2$- and GSO projections, that is, $(+,+)$, $(-,-)$, $(+,-)$, $(-,+)$. The choices $(+,+)$, $(-,-)$ lead to $\mathcal{N}=3$ superconformal algebras in space-time. These spin fields S^a_r belong to representation $(\mathbf{2},\mathbf{3})$ in the $\mathrm{SL}(2;\mathbb{R})\times\mathrm{SU}_D(2)$. But in the remaining two cases, possible spin operators belong to $(\mathbf{2},\mathbf{1})$ representation of $\mathrm{SL}(2;\mathbb{R})\times\mathrm{SU}_D(2)$ and only $\mathcal{N}=1$ algebras are retained. They satisfy $\mathcal{N}=1$ superconformal algebra (SCA) combined with Virasoro generators.

In the context of the IIA string theory, the chiralities of supercharges are opposite in space-time and the GSO projections act on left-, right-moving parts with different signatures. The resulting type IIA theory has an $(\mathcal{N},\overline{\mathcal{N}})=(3,1)$ superconformal symmetry in space-time with an extra affine SU(2) algebra in the $\overline{\mathcal{N}}=1$ part. The $\overline{\mathcal{N}}=1$ part has a degree of freedom of the diagonal S^3_D, that is, $\mathrm{SU}_D(2)$ affine symmetry in space-time. We construct modes of a set of these $\overline{\mathcal{N}}=1$ affine $\mathrm{SU}_D(2)$ generators explicitly. The precise physical meaning of this remains to be understood.

Also we present physical vertex operators, in particular, chiral primary fields in the $\mathcal{N}=3$ SCFT. We study short multiplets of (chiral part of) the $\mathcal{N}=3$ theory. It has $\mathcal{N}=1$ doublet states and we identify (integrated form of) $Q^-_{-1/2}|\tilde{h},2\tilde{h}\rangle$s with marginal operators. A candidate for the space-time $(\mathcal{N},\overline{\mathcal{N}})=(3,1)$ full theory is a orbifold-type sigma model like some symmetric product (more precisely Hilbert schemes) of a compact space. In order to establish the AdS_3/CFT_2 correspondence, we have to consider the total Fock space of the space-time CFT and clarify the properties of the space-time vacua.

Next, from the point of view of worldsheet theory, we speculate about the roles of the currents J^3, K^3 and about relations between $N=2$ $CP^{k_{so}}$-model and AdS_3-model. There are several integrable deformations in the context of the CFT. For an example, we can deform the theory by the most relevant operator associated with a Kähler form of the CP model. As another example, one can obtain Toda-type theories by using Chebyshev polynomials and then their solitons connect different vacua.

In the viewpoint of M-theory, we present a brane configuration whose near horizon geometry describes the AdS_3 background. The geometry is obtained at the throat limit of two sets of M5-branes intersecting in one direction, together with M2-branes with a fixed-plane extending to oblique directions. The two kinds of M5-

branes ($M5^{(1)}$, $M5^{(2)}$) are mirror images one another. Then the $\mathbb{Z}_2$-group acts on a classical solution of the M2, $M5^{(1)}$, $M5^{(2)}$ system. By taking a large charge limit, we can obtain a near horizon geometry $AdS_3 \times (S^{3(1)} \times S^{3(2)} \times \mathbb{R})/\mathbb{Z}_2 \times \mathbb{R}$ with a natural $\mathbb{Z}_2$ action ($S^{3(1)} \leftrightarrow S^{3(2)}$, $\mathbb{R} \leftrightarrow -\mathbb{R}$). By shrinking the radius of a longitudinal direction $\mathbb{R}$, we construct the orbifold-type space with the $\mathbb{Z}_2$-action in the IIA theory.

The M2 brane ends on the M5 and the intersecting part is a $(1+1)$-dimensional object. When one compactifies one longitudinal direction (in stringy limit), it is known as a little string trapped in the NS5-brane. In our set-up, conformal theory lives on the fixed plane in the background with solitonic NS5-branes and little strings. We might have a new exotic low energy susy gauge theory with some extra multiplets. The near horizon region in this model is the same as an infrared limit for this gauge theory. It is a challenging problem to analyse these in the M(atrix) model.

In this paper, for simplicity, we consider only a $\mathbb{Z}_2$-action. However we would expect that the method developed here is applicable to other general backgrounds with actions of discrete groups. We would like to discuss these subjects elsewhere.

Acknowledgments

K.S. and S.Y. are grateful to T. Uematsu for valuable discussions and useful comments. Y.I. gratefully acknowledges helpful conversations with S. Matsuda. The authors also thank the Yukawa Institute for Theoretical Physics for hospitality and the participants of the associated seminars for discussions. K.S. is supported in part by the Grant-in-Aid for Scientific Research from the Ministry of Education, Science, Sports and Culture 10740117. S.Y. is a Research Fellow of the Japan Society for the Promotion of Science and is supported in part by the Grant-in-Aid for Scientific Research from the Ministry of Education, Science, Sports and Culture.

A. SO(10) gamma matrices

Here we summarize the SO(10) Gamma matrices.

We choose a convention for Gamma matrices Γ^M, $(M = 1, \ldots, 10)$

$$
\begin{aligned}
\Gamma^1 &= \sigma^1 \otimes \mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1} & \Gamma^6 &= \sigma^3 \otimes \sigma^3 \otimes \sigma^2 \otimes \mathbf{1} \otimes \mathbf{1} \\
\Gamma^2 &= \sigma^2 \otimes \mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1} & \Gamma^7 &= \sigma^3 \otimes \sigma^3 \otimes \sigma^3 \otimes \sigma^1 \otimes \mathbf{1} \\
\Gamma^3 &= \sigma^3 \otimes \sigma^1 \otimes \mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1} & \Gamma^8 &= \sigma^3 \otimes \sigma^3 \otimes \sigma^3 \otimes \sigma^2 \otimes \mathbf{1} \\
\Gamma^4 &= \sigma^3 \otimes \sigma^2 \otimes \mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1} & \Gamma^9 &= \sigma^3 \otimes \sigma^3 \otimes \sigma^3 \otimes \sigma^3 \otimes \sigma^1 \\
\Gamma^5 &= \sigma^3 \otimes \sigma^3 \otimes \sigma^1 \otimes \mathbf{1} \otimes \mathbf{1} & \Gamma^{10} &= \sigma^3 \otimes \sigma^3 \otimes \sigma^3 \otimes \sigma^3 \otimes \sigma^2 \,,
\end{aligned}
$$

where the σ^a, $(a = 1, 2, 3)$ are the Pauli matrices, and $\mathbf{1}$ is the 2×2 unit matrix described as

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \mathbf{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

These Γ^M's satisfy anti-commutation relations $\{\Gamma^M, \Gamma^N\} = 2\delta^{MN}$. The chirality operator Γ^{11} is expressed as

$$\Gamma^{11} = -i\Gamma^1\Gamma^2 \cdots \Gamma^{10} = \sigma^3 \otimes \sigma^3 \otimes \sigma^3 \otimes \sigma^3 \otimes \sigma^3.$$

Also we define a unitary matrix C representing a charge conjugation operation as

$$C = \sigma^2 \otimes \sigma^1 \otimes \sigma^2 \otimes \sigma^1 \otimes \sigma^2 \,.$$

This C satisfies following several relations

$$C\Gamma^M C^{-1} = -(\Gamma^M)^T, \qquad C = C^\dagger = C^{-1} = -C^T \,.$$

Then the $C\Gamma^M$ is symmetric as a matrix.

In this paper, these representations of Gamma matrices appear as cocycle factors in the the following OPEs including spin fields

$$\begin{aligned} \boldsymbol{\psi}^M(z) S_\alpha(w) &= \frac{1}{\sqrt{2}(z-w)^{1/2}} S_\beta (\Gamma^M)^\beta{}_\alpha \,, \\ e^{-\varphi/2} S_\alpha(z) e^{-\varphi/2} S_\beta(w) &\sim \frac{1}{z-w} e^{-\varphi} (C\Gamma^M)_{\alpha\beta} \boldsymbol{\psi}^M \,. \end{aligned}$$

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JHEP02(1999)026